



K¹² MATH WHITE PAPER

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K¹² Math is designed to enable every student to succeed in math. The program is based on extensive research in math education and cognitive science, the content standards of all fifty states, and the recommendations of highly respected professional organizations. In addition to reflecting our commitment to research, the K¹² curriculum exhibits a deep understanding of what children need to become both mathematically proficient and capable, productive adults.

By offering each student countless opportunities to learn and understand significant mathematical concepts and procedures, K¹² Math exceeds the benchmarks of many states. The program helps students develop deep, meaningful insight into numbers and operations and a common-sense and practical approach to using them. Through K¹² Math, children are able to distinguish and understand diverse kinds and representations of numbers and to use them in wide-ranging conditions. In addition to developing fluent and flexible computational skills, students learn the justifications and meanings of mathematical procedures, making it possible to apply the procedures to an extensive range of problems and situations. We use technology in a way that maximizes its unique powers—to provide a broad array of resources to students, teachers, and parents, including animated models of mathematical concepts and procedures, simulations of real-life situations, and teaching games—and to tailor the curriculum to each student’s and class’s learning needs.

In both its online and offline forms, the program provides an active and engaging, multi-sensory approach promoting students’ understanding of the concrete realities underlying mathematical concepts. Regular practice and review ensure mastery of basic skills; online games and animations motivate students and illustrate concepts; and challenge problems help develop critical-thinking skills.

A Research-based Curriculum

What makes K¹² Math unique is its heavy foundation in cognitive science research. To design the curriculum, we have drawn extensively on several major areas of research:

- How children learn math
- The nature of mathematical knowledge
- The most powerful online and offline instructional strategies.

We know, for example, that the knowledge of advanced learners is highly organized around fundamental principles, or “big ideas.” To help students organize their own mathematical thinking, our curriculum is structured around big ideas that we have identified by working with mathematicians and expert math educators. Students are able to see how everything they learn has meaning and connects to the “big picture” of math. Learning math in this way provides a more solid foundation for future learning than that provided by a traditional curriculum. It also allows students to achieve the mastery needed to succeed on state tests.

In addition to drawing on existing research, K¹² conducts its own ongoing research on learning, instruction, curriculum design, and effective uses of technology. We analyze state test scores and other measures of student learning and program effectiveness, conduct experimental studies of our curriculum and its components, and continually develop and test enhancements to the program. Our ongoing research ensures that we not only maintain a superior curriculum but that we continually fine-tune and improve it.

In keeping with findings from research on learning and instruction, K¹² Math balances several critical principles:

- Effective math programs provide the necessary basic skills upon which more challenging and

- complex mathematical concepts can be built.
- Early and systematic instruction, with consistent review and practice of basic skills and the early introduction of problem-solving strategies, supports sound conceptual understanding that transitions from concrete representation to abstract symbolic notation.
- Conceptual understanding is the key to successful problem solving, long-term learning, and mastery of advanced subjects such as algebra and calculus.

Building Skills, Knowledge, and Understanding

Consistent with math education research, the K¹² approach recognizes the importance of several kinds of learning: practice of basic skills until they have been mastered; memorization of algorithms, formulas, and facts; and the understanding of big ideas and other concepts that give meaning to skills, facts, and problem solving. To become more proficient in math, students have to memorize basic facts, learn how and when to use specific algorithms, formulas, and problem solving strategies, and develop an understanding of the fundamental conceptual structures of math.

Recognizing that mastery of math involves a myriad of proficiencies, we have built into the curriculum a wide range of expectations for students. These expectations include:

- Problem solving—the principal reason for studying math—posing questions; analyzing, translating and illustrating results; drawing diagrams; using trial and error; applying rules of logic; recognizing relevant facts; and scrutinizing conclusions
- The application of math to everyday situations, which require students to translate mathematical relationships into models that must be solved, interpreted, and applied to daily activities
- The ability to estimate, approximate and determine the reasonableness of answers—necessary skills for estimating quantity, length, distance, weight, etc., and for students to know when a result is precise enough for the purpose at hand
- Appropriate computational skills—knowledge of single digit number facts and mental arithmetic, including addition, subtraction, multiplication, division with whole numbers, decimals, fractions, percents, algebraic computation, and complicated computations is necessary
- Concepts of point, line, plane, parallel, perpendicular, and basic properties of simple geometric figures, with emphasis on measurement and problem solving; recognizing similarities and differences among objects as the underpinnings for formal geometry
- Understanding measurements, including distance, weight, time, capacity, temperature, and angles; calculating simple areas and volumes; and using both metric and customary systems with appropriate tools
- Reading, interpreting, and constructing tables, charts, and graphs, including analyzing numerical information and partitioning this information into manageable/meaningful terms, and the ability to use conclusions with tables, maps, charts and graphs
- Using mathematical models to predict and determine the likelihood of future events and identifying immediate past experiences that do not affect the likelihood of future events
- Understanding computer capabilities—what computers can and cannot do.

Metacognition

Metacognition refers to the process of planning, reflecting on, and evaluating one's own learning. Research has consistently demonstrated that metacognition is a characteristic of good learners, and plays a vital role in successful math learning. Recognizing this, the K¹² math curriculum supports the development of metacognition in a variety of ways. For example, students learn how to think about

their own learning, monitor their understanding of key concepts, and evaluate their overall progress. When solving math problems, students are regularly prompted to monitor their understanding of each problem and think about procedures that lead them toward solutions. Since there is a wide range of knowledge, skills, and learning preferences at each grade level, students are individually guided to identify the learning and study strategies that work best for them.

The K¹² math curriculum features:

- A carefully structured sequence of lessons designed to help students steadily progress from fundamental skills to more complex mathematical concepts
- A multi-sensory, hands-on approach to ensure that children understand the concrete realities and underpinnings of mathematical concepts (e.g., shape, length, quantity)
- Regular practice, review, and assessment to ensure mastery of basic skills
- Online games and animations that help illustrate concepts, motivate, and instruct

For more information about the K¹² Math program and how this curriculum could be used by your school or district, please contact 866.903.5122 or visit K12.com/educators.

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Online Resources

Ross, David. The math wars. <http://www.objectivistcenter.org/showcontent.aspx?ct=245&h=53>

National Council of Teachers of Math, A research companion to Principles and Standards for School Math. <http://nctm.org/>

National Council of Teachers of Math, Curriculum Focal Points for Pre-Kindergarten through Grade 8 Math. <http://nctm.org/>

Hayes, N. <http://www.newhorizons.org/trans/hayes%202.htm>

